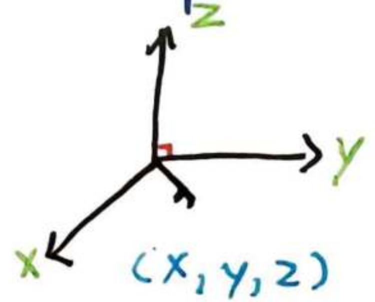
12.1 Coordinate in 3-Space

3-Space ($\mathbb{R}^3$) | 2-Space ($\mathbb{R}^2$)

$\in$ ↑ "in" | $:$ ↑ "such that"



(x, y, z)

(x, y)

I. Coordinate Planes

A Coordinate plane is the set of pts. in which a specified pt. is 0 (

Ex. The ~~x~~y-plane (aka the $z=0$) in $\mathbb{R}^3$ is $\Pi = \{P = (x,y,z) \in \mathbb{R}^3 : z = 0\}$,

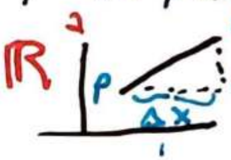
yz-plane in $\mathbb{R}^3$ $\{P = (x,y,z) \in \mathbb{R}^3 : x = 0\}$, xz-plane in $\mathbb{R}^3$ $\{P = (x,y,z) \in \mathbb{R}^3 : y = 0\}$

Aside: Distances

$\mathbb{R}^3$

$P = (x_0, y_0, z_0)$ length $= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$

$\mathbb{R}^2$



$d(P,Q) = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

$d(P,Q) = \sqrt{\left(\sqrt{(x_1-x_0)^2+(y_1-y_0)^2}\right)^2 + (z_1 - z_0)^2}$

$= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$ ⟵ The Distance Formula ($\mathbb{R}^3$)

II. Spheres

Let $r > 0$ and let $P \in \mathbb{R}^3$. The spere of radius centered at P is $S = \{Q \in \mathbb{R}^3 : d(P,Q) = r\}$

If P has coordinates $P = (x_0, y_0, z_0)$, then $S = \{Q \in \mathbb{R}^3 : d(P,Q) = r\} = \{(x_1, y_1, z_1) \in \mathbb{R}^3 : \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2} = r\}$

$= \{x_1, y_1, z_1\} \in \mathbb{R}^3 : (x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 = r^2$

not solid

NB: spheres are "surface" of a hollow ball

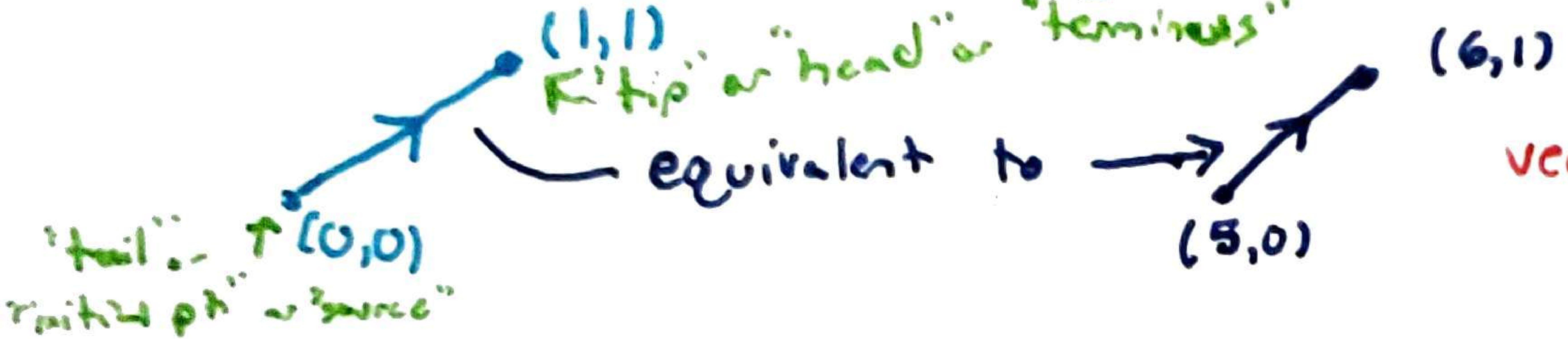
solid ball is defined by $(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 \leq r^2$

NB: everything we've done so far has analogues in higher dimentions. E.g., there is $\mathbb{R}^4$ w/ a distance formula $\{\mathbb{R}^4 (x, y, z, w) : x, y, z, w \in \mathbb{R}\}$

$P(x_1, y_1, z_1, w_1), Q(x_0, y_0, z_0, w_0)$ $d(P,Q) = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2 + (w_1 - w_0)^2}$

12.2: Vectors

Def: A vector in $\mathbb{R}^3$ is a directed line segment, where two vectors are equivalent when they are linear shifts



vectors parametric?